

# Petascale Simulations of Type Ia Supernovae



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Galaxy NGC 4526 imaged by the Hubble Space Telescope ([www.nasa.gov](http://www.nasa.gov))

60 million light years away; located in the Virgo constellation



SN1994D (Type Ia supernova)

The supernova is as bright as the host galaxy!



# Outline

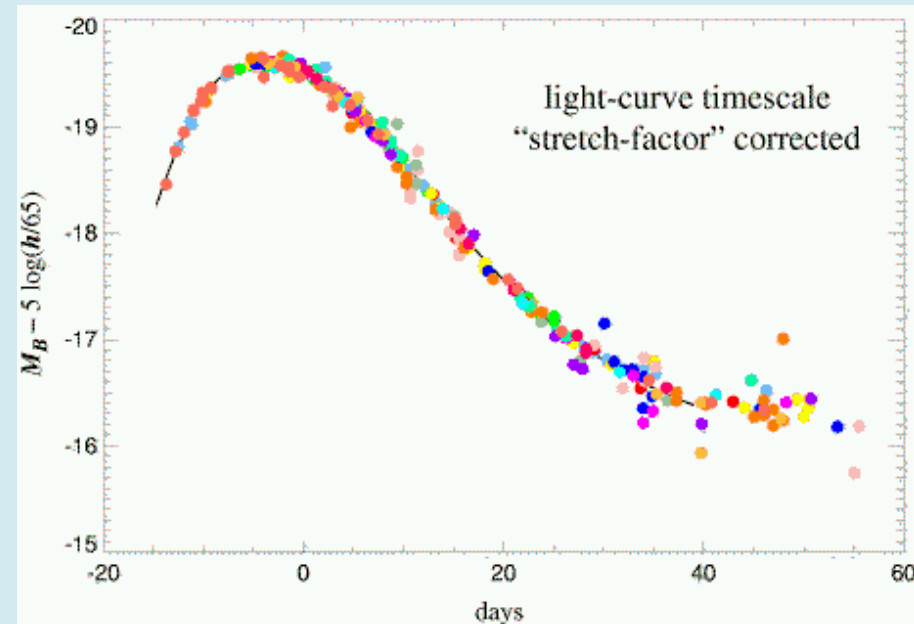
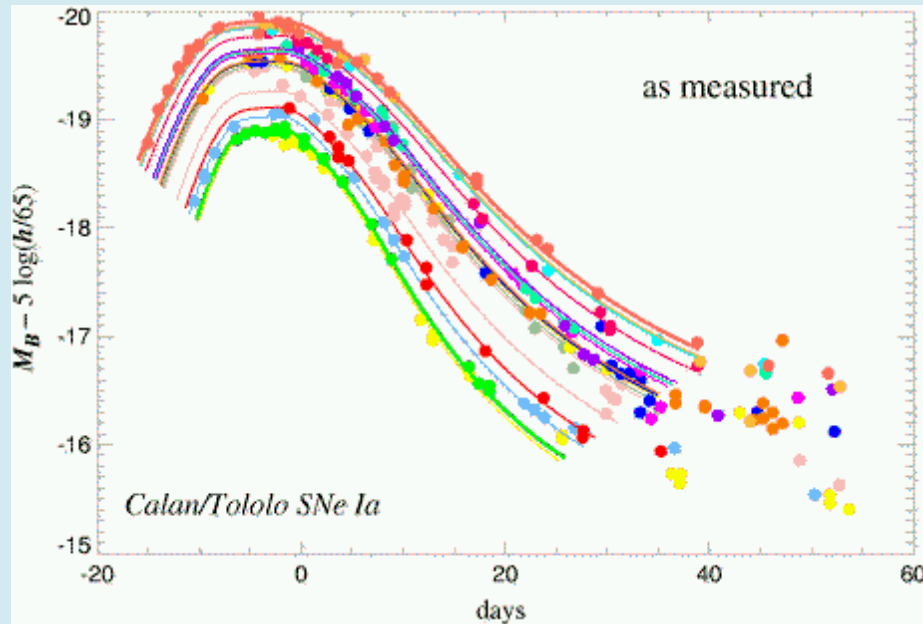
- Motivation: Type Ia Supernovae
- MAESTRO: Low Mach Number Astrophysical Solver
  - Mathematical formulation to exploit time scales of interest
  - Adaptive Mesh Refinement (AMR)
  - Massively parallel (100,000 cores) implementation
- Primary Collaborators
  - Ann Almgren, John Bell, Mike Lijewski, Candace Gilet: LBNL
  - Mike Zingale, Chris Malone: Stony Brook University



- Using modern telescopes, Type Ia supernova light curves can now be observed several hundred times per year:
  - Spectra contains silicon, lacks hydrogen
  - Peak powered by radioactive decay of nickel

# Type Ia Supernovae are Distance Indicators

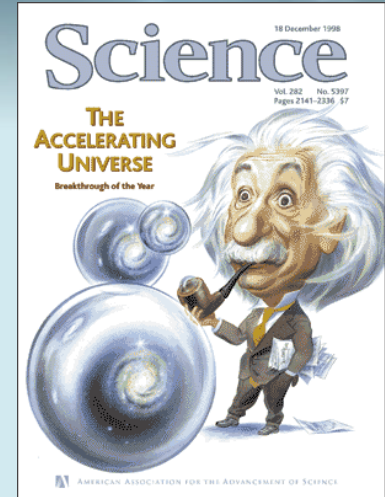
- By observing Type Ia supernovae at known, nearby distances, scientists have established a width-luminosity relationship; **wider = brighter**.



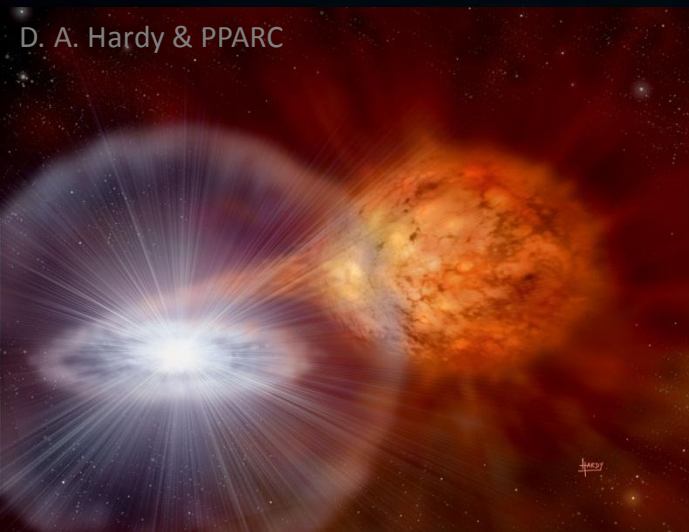
- Theory: by observing the peak luminosity and decay rate, we can determine the **distance** to a host galaxy.
  - Particularly useful for mapping distant galaxies since they are so bright!

# Type Ia Supernovae are Speed Indicators

- Due to the observed redshift, we know the **speed** at which the host galaxy is moving away from us.
  - Led to discovery of the acceleration of the expansion of the universe (1998)
- Problem: We don't know how well the width-luminosity relationship holds for distant Type Ia supernovae
  - Farther away = earlier in the life of the universe
  - Composition of stars was different back then...
  - Not even sure if accepted models properly describe nearby events...
- How will we study this problem using computers?

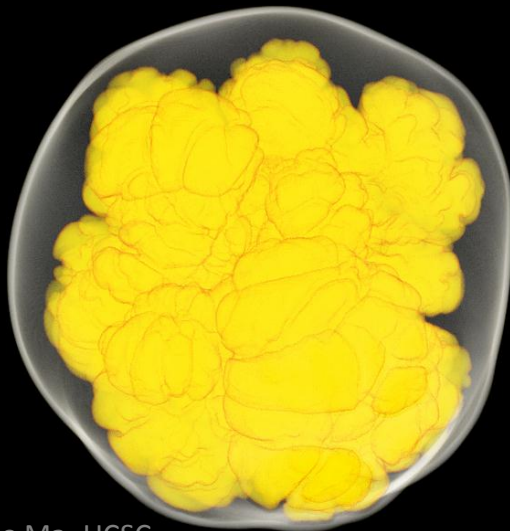
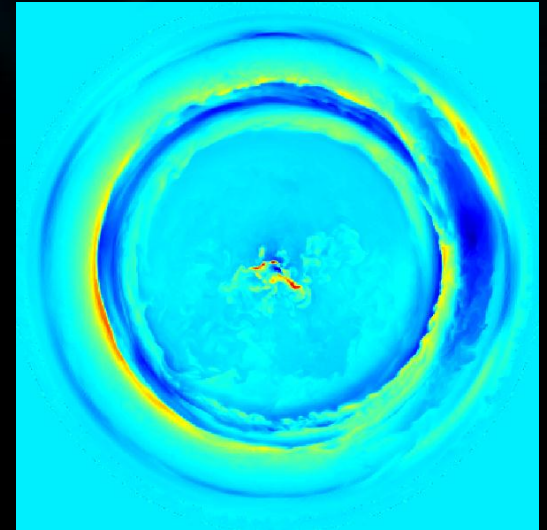


# The Phases of Type Ia Supernovae: Single Degenerate Model



A white dwarf accretes matter from a binary companion over **millions of years**.

Smoldering phase characterized by subsonic convection and gradual temperature rise lasts **hundreds of years**.



Flame (possibly) transitions to a detonation, causing the star to explode within **two seconds**.

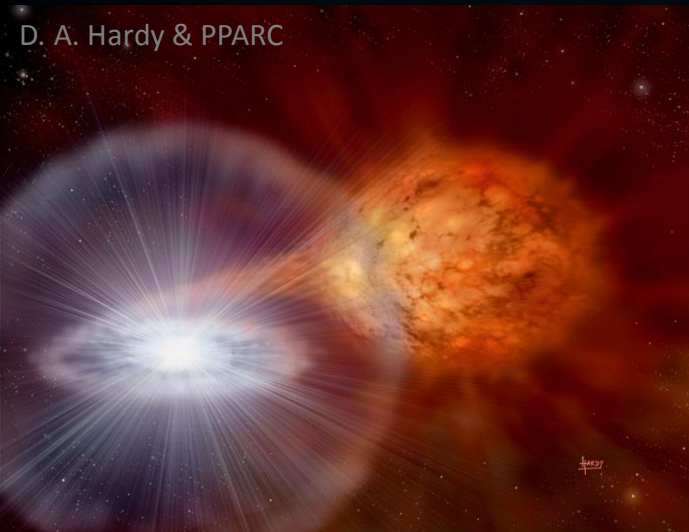
The resulting event is visible from Earth for **weeks to months**.



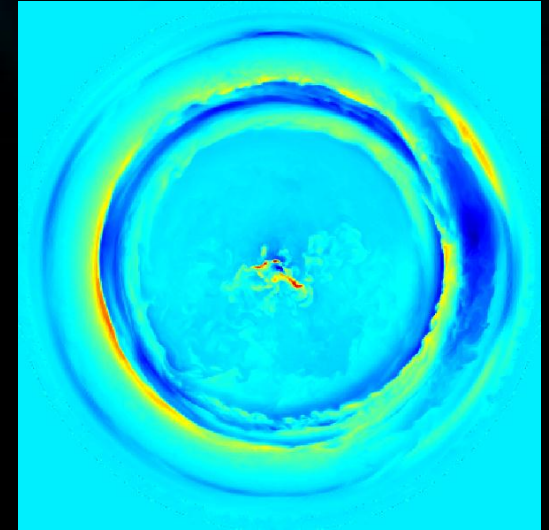
SN 1994D (High-Z SN Search team)



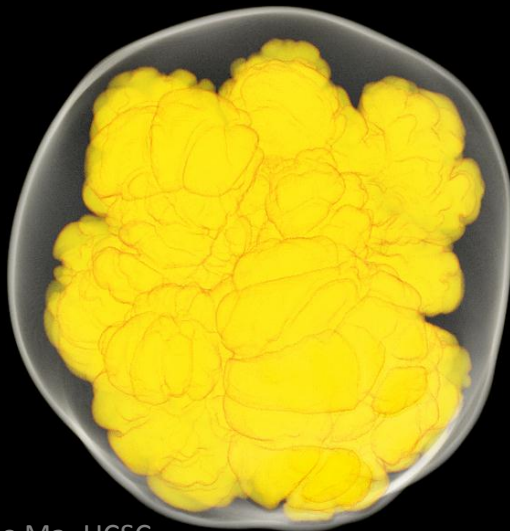
# Each Phase has Different Computational Requirements



KEPLER  
(Woosley, UCSC)



MAESTRO  
(LBNL)



CASTRO (LBNL)

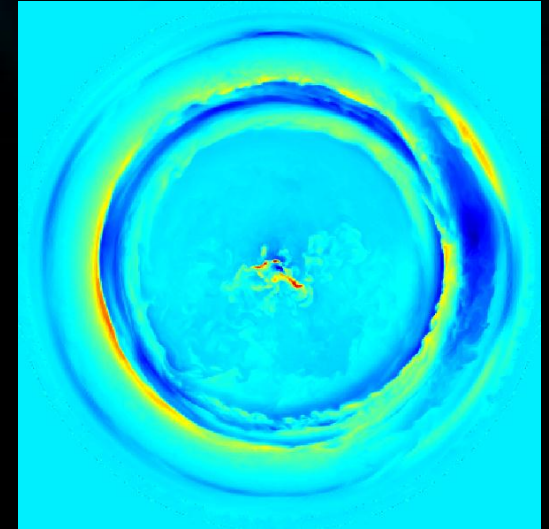


SEDONA  
(Kasen, Nugent, Thomas, LBNL)



# Computing the Convective Phase

MAESTRO  
(LBNL)



- In this talk we will focus on the last few hours of convection preceding ignition.
  - We wish to use MAESTRO to determine the initial conditions for the detonation / explosion phase for CASTRO
    - Previous studies have artificially seeded hot ignition points into their initial conditions
  - Low Mach number regime;  $M = U/c$  is  $O(10^{-2})$
  - Long-time integration infeasible using fully compressible approach



# **MAESTRO: Low Mach Number Astrophysics**

## **- Algorithmic Details**

# What is MAESTRO?

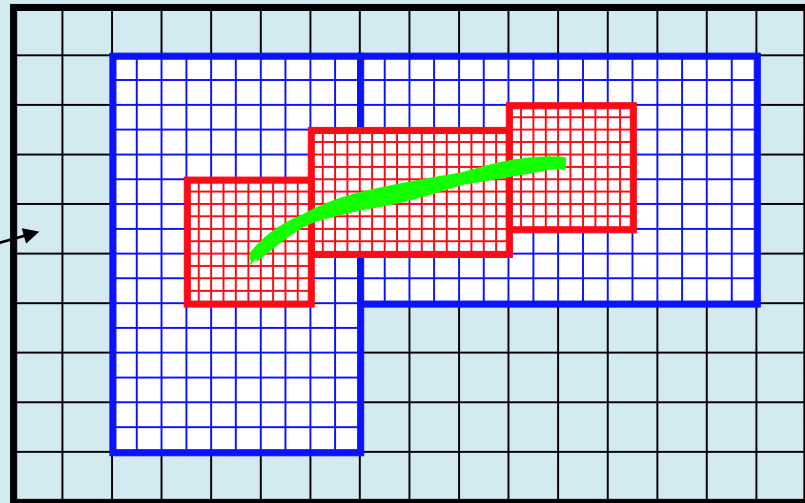
- Key Theme
  - Having access to hundreds of thousands of CPUs is essential, but still not enough to solve this problem. We must also utilize a special mathematical formulation as well as AMR technology to address this problem.



# What is MAESTRO?

- MAESTRO is a massively parallel, finite volume, AMR code for low Mach number astrophysical flows
  - Massively Parallel: Scales to 100,000 cores
  - Finite Volume: Solution in each Cartesian cell represents the average over the cell
  - AMR: Block-structured approach with logically rectangular grids

$\rho, \mathbf{u}, p, T$ , etc.



- Low Mach Number: Fluid speed is small compared to the speed of sound
- Astrophysical Flows: Modular equation of state and reaction networks

# Low Mach Number Equation Set

- Equation set derived using low Mach number asymptotics
  - Mach number:  $M = U/c$
  - Looks similar to the standard equations of compressible flow, but sound waves have been analytically removed
    - Enables time steps constrained by the fluid velocity CFL, not the sound speed CFL:

$$\Delta t_{\text{compressible}} < \frac{\Delta x}{|u| + c} \qquad \Delta t_{\text{lowMach}} < \frac{\Delta x}{|u|}$$

- Low Mach time step is a factor of  $1/M$  larger than a compressible time step, enabling long-time integration!

# Low Mach Number Equation Set

- Derived from fully compressible equation set

$$\frac{\partial(\rho X_k)}{\partial t} = -\nabla \cdot (\rho X_k \mathbf{u}) + \rho \dot{\omega}_k \quad \text{conservation of mass}$$

$$\frac{\partial(\rho \mathbf{u})}{\partial t} = -\nabla \cdot (\rho \mathbf{u} \mathbf{u}) - \nabla \pi + \rho \mathbf{g} \quad \text{conservation of momentum}$$

$$\frac{\partial(\rho h)}{\partial t} = -\nabla \cdot (\rho h \mathbf{u}) + \rho H \quad \text{conservation of energy}$$

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$\rho$  density

$\mathbf{u}$  velocity

$X_k$  mass fraction of species “k”

$\dot{\omega}_k$  reaction rate of species “k”

$h$  specific enthalpy

$\mathbf{g}$  gravity

$H$  reaction heating

$\pi$  deviation from ambient pressure



# Low Mach Number Equation Set

- Our system is closed with an equation of state, which keeps system in thermodynamic equilibrium.
  - Differentiate equation of state along particle paths to represent as a divergence constraint:

$$\nabla \cdot (\beta_0 \mathbf{u}) = \beta_0 S$$

$\beta_0 \rightarrow$  captures expansion/contraction of fluid due to changes in altitude

$S \rightarrow$  captures local compressibility effects due to reactions and thermal diffusion

- Numerical enforcement of divergence constraint analogous to solution methodology for incompressible flow
  - Pressure-projection method involving a variable-coefficient Poisson solve

# Numerical Methodology

- Strang splitting couples advection/reaction/diffusion
  - **Advection** using Godunov approach
  - **Reactions** using stiff ODE solver
  - **Diffusion** semi-implicit (multigrid)
  - **Divergence-constraint** requires elliptic solve (multigrid)

$$\frac{\partial(\rho X_k)}{\partial t} = -\nabla \cdot (\rho X_k \mathbf{u}) + \rho \dot{\omega}_k$$

$$\frac{\partial(\rho \mathbf{u})}{\partial t} = -\nabla \cdot (\rho \mathbf{u} \mathbf{u}) - \nabla \pi + \rho \mathbf{g}$$

$$\frac{\partial(\rho h)}{\partial t} = -\nabla \cdot (\rho h \mathbf{u}) + \nabla \cdot \kappa \nabla T + \rho H$$

$$\nabla \cdot (\beta_0 \mathbf{u}) = \beta_0 S$$

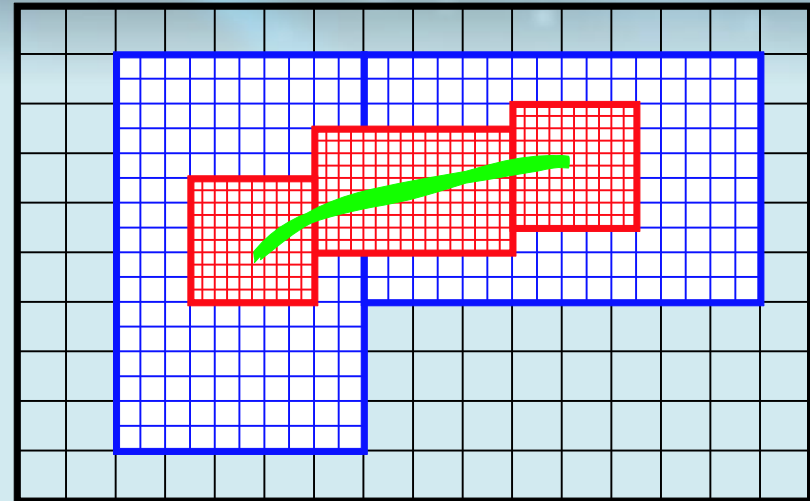
# Computational Efficiency

- In our white dwarf simulations, the Mach number is approximately  $M \approx 0.05$ 
  - The low Mach number time step is a factor of **70** greater than a compressible time step
  - However, a low Mach number time step takes approximately 2.5 times longer to compute, mostly due to the linear solvers (multigrid)
- Thus, to advance the solution to the final time, **MAESTRO is a factor of  $(70 / 2.5) \approx 28$  more efficient than a compressible algorithm**, given the same number of computational resources for this problem



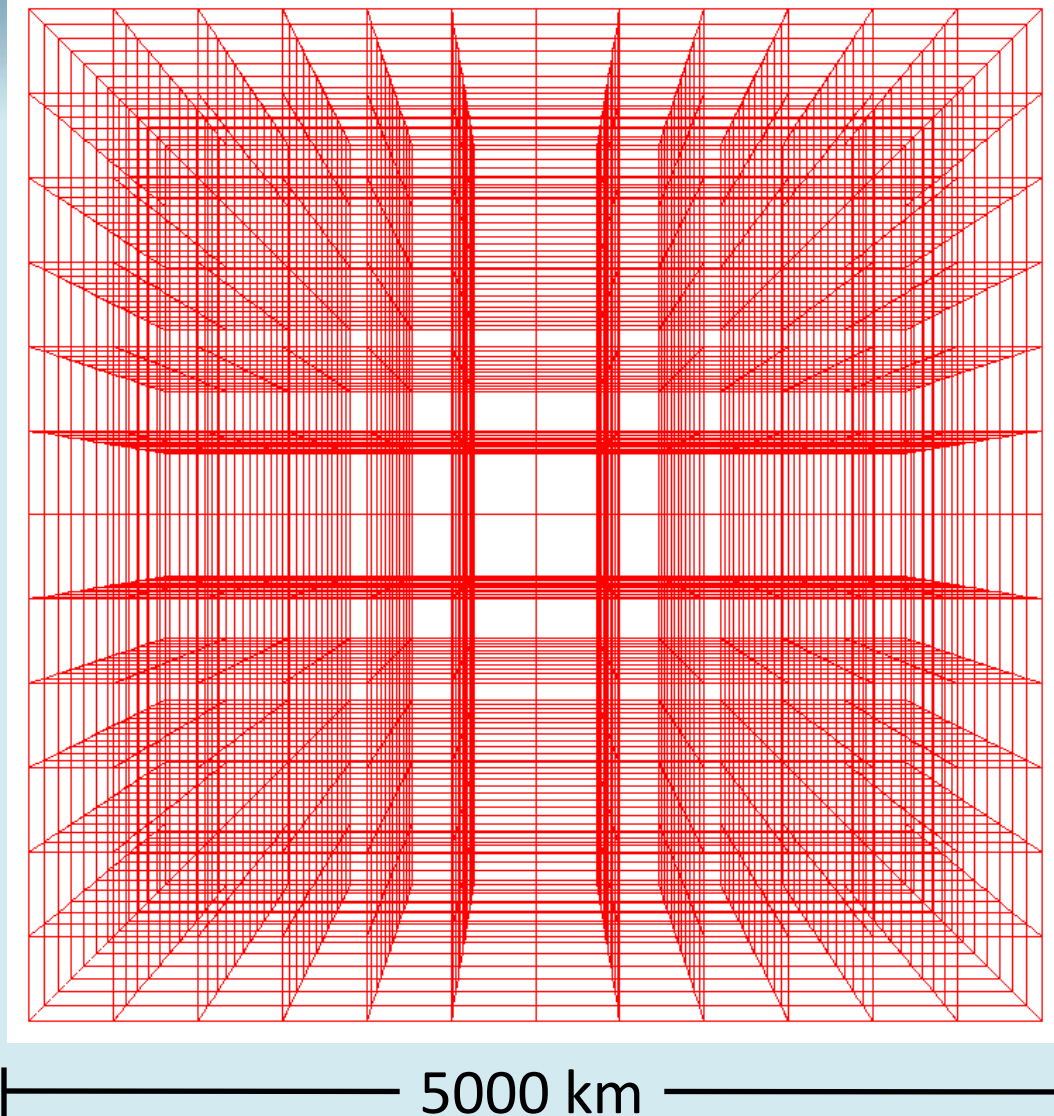
# Adaptive Mesh Refinement

- Incorporate AMR using established techniques
  - Advance each level independently and synchronize fluxes, velocities, and pressure at coarse-fine interfaces
- For the full star problem, we need to consider our tagging criteria
  - Burning occurs near core, driving flow in the inner-convective region of the star.
  - We expect ignition point(s) to be near the center of the star



# Adaptive Mesh Refinement

- $576^3$  (8.7 km)
  - $1728 \cdot 48^3$  grids
  - 191 Million Cells

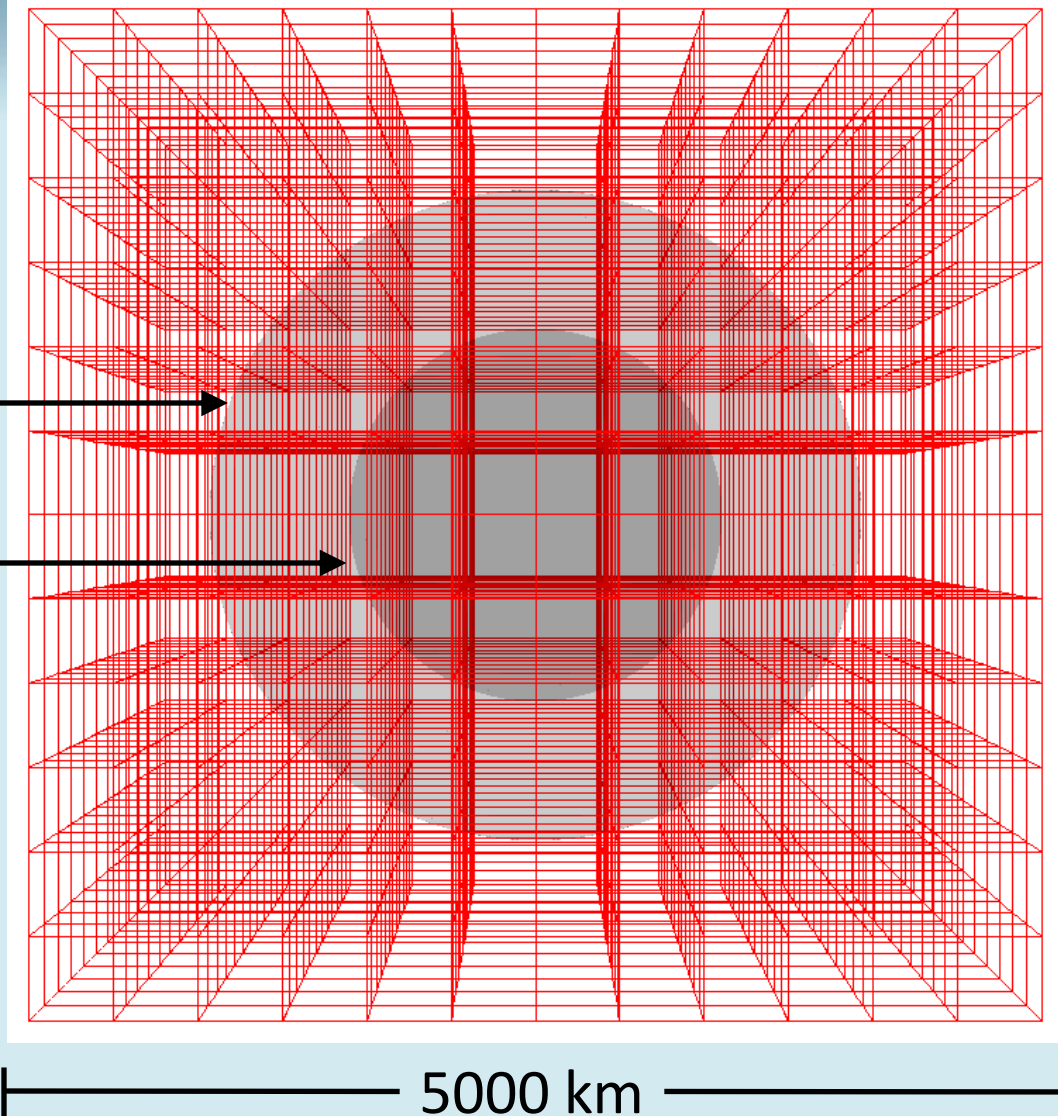


# Adaptive Mesh Refinement

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Edge of Star

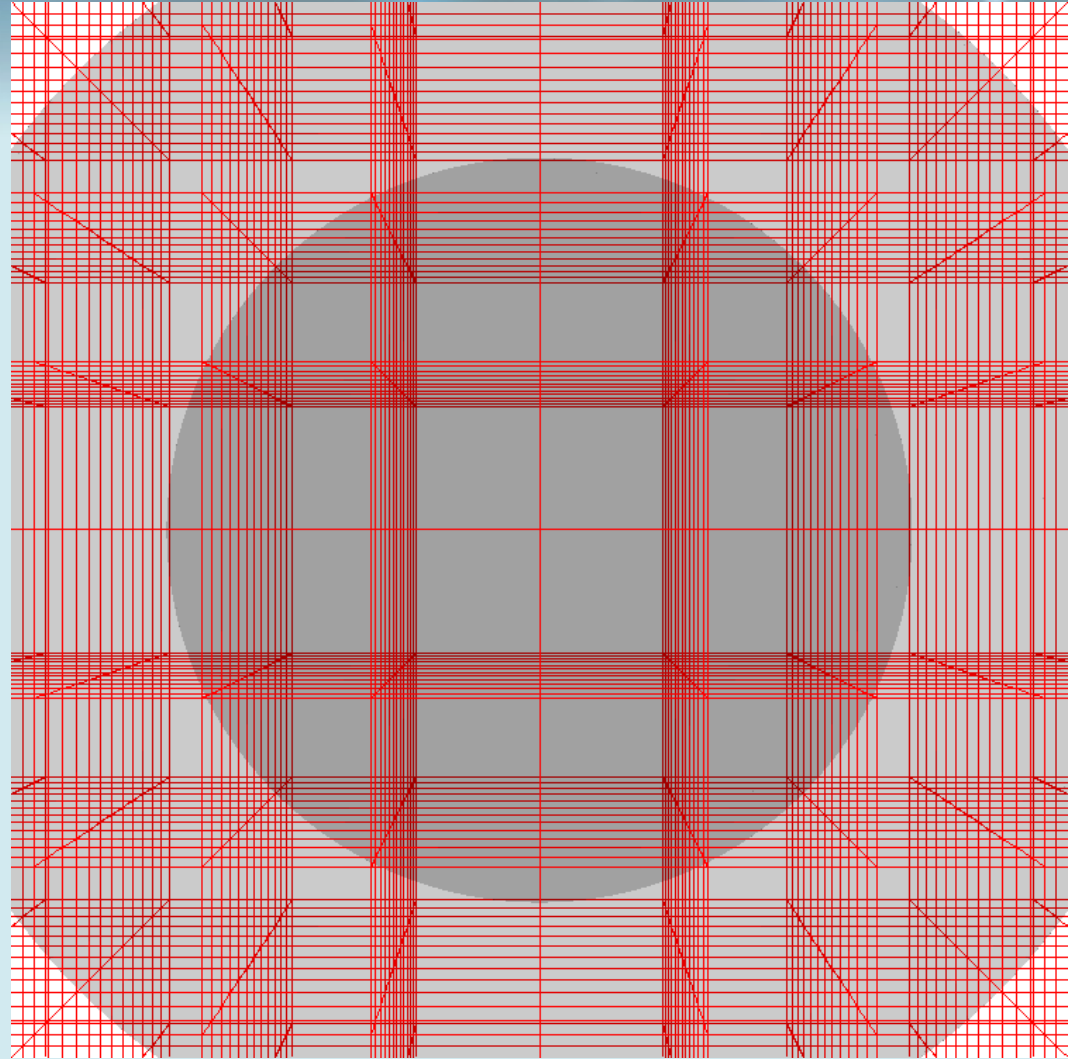
Convective Zone  
Boundary





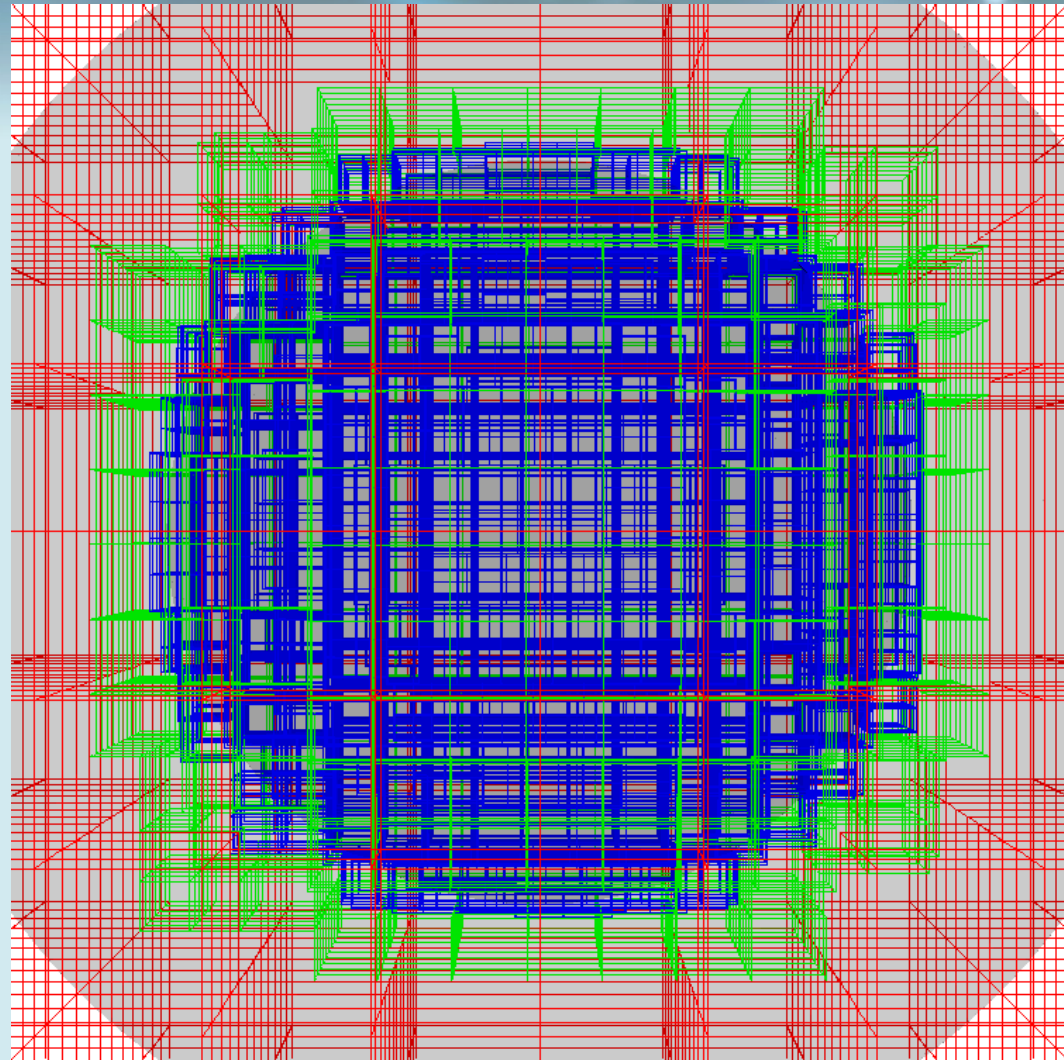
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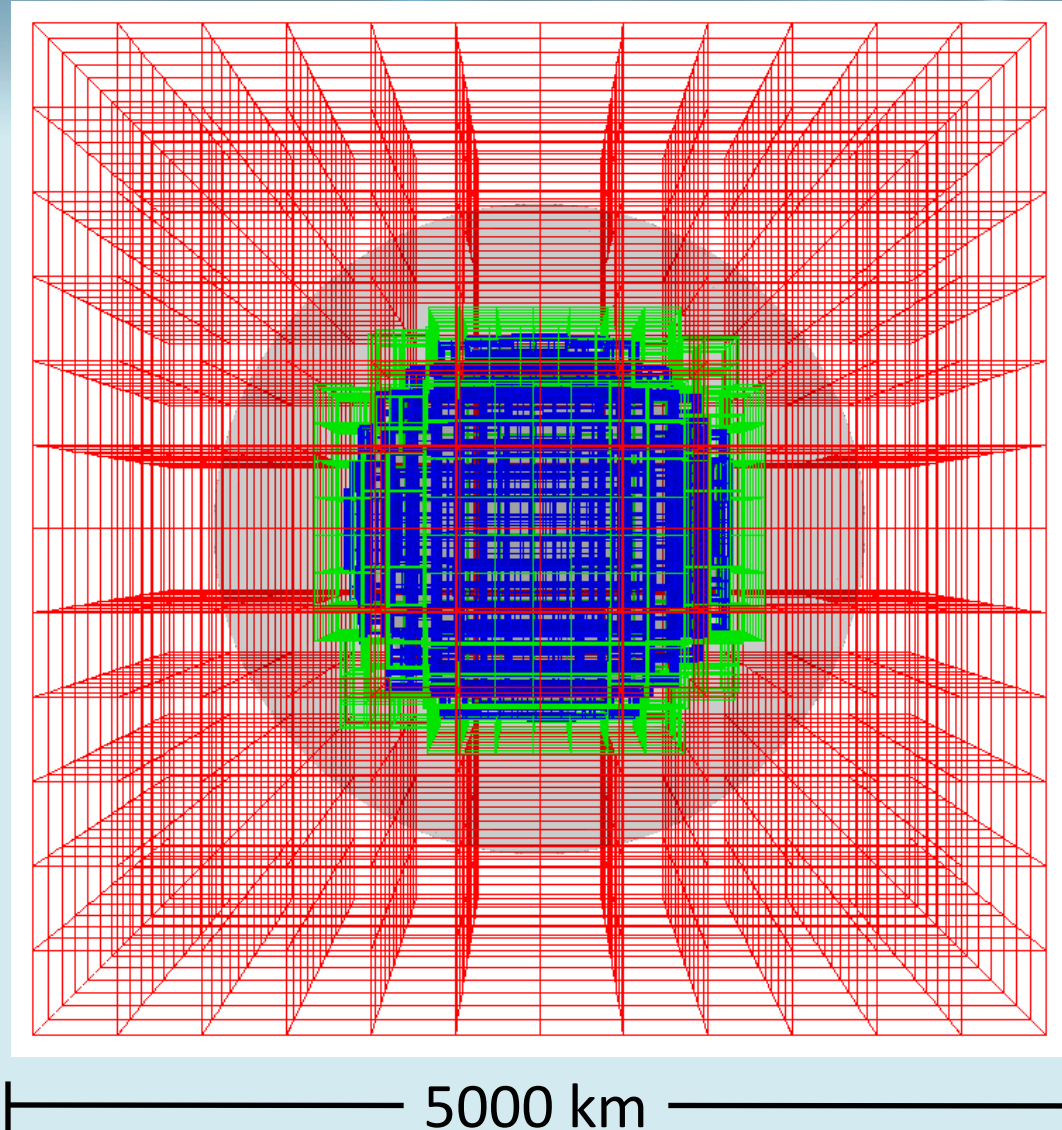
# Adaptive Mesh Refinement

- $576^3$  (8.7 km)
  - $1728 \cdot 48^3$  grids
  - 191 million cells
- $1152^3$  (4.3 km)
  - 1684 grids
  - 148 million cells
  - 9.7% of domain
- $2304^3$  (2.2 km)
  - 3604 grids
  - 664 million cells
  - 5.4% of domain



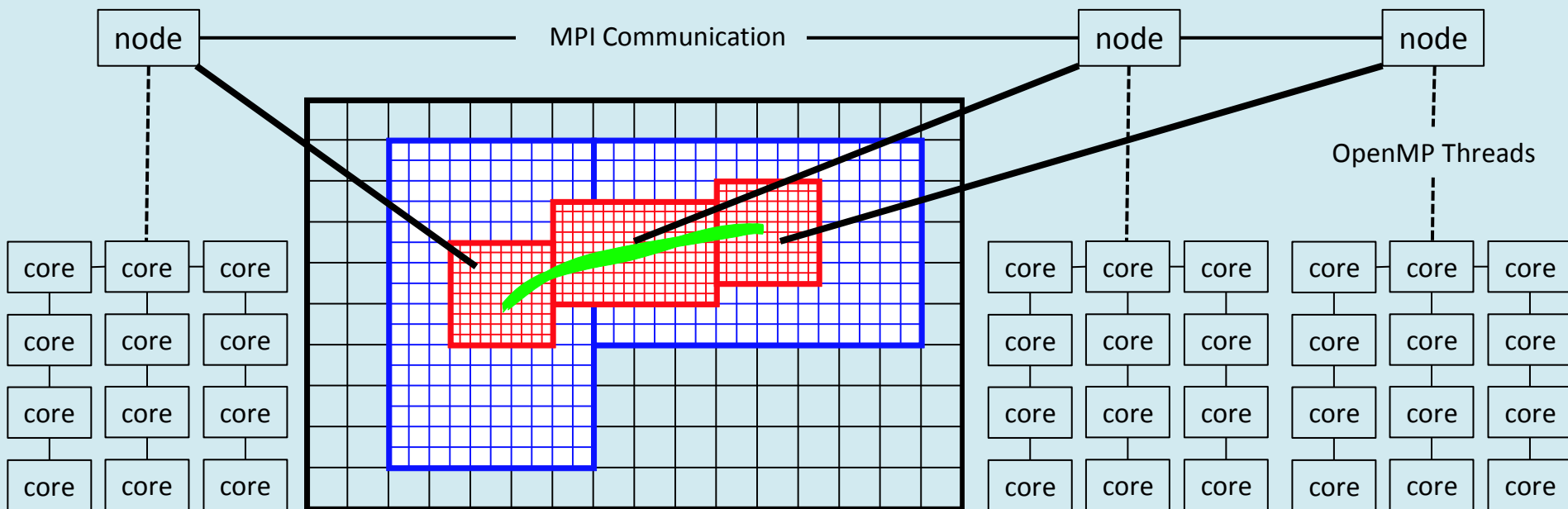
# Adaptive Mesh Refinement

- A  $2304^3$  simulation with no AMR would contain 12.2 billion cells.
- Our simulation contains a total of 1.0 billion cells, requiring a **factor of 12 less work**.
  - Excluding AMR overhead, which is several percent



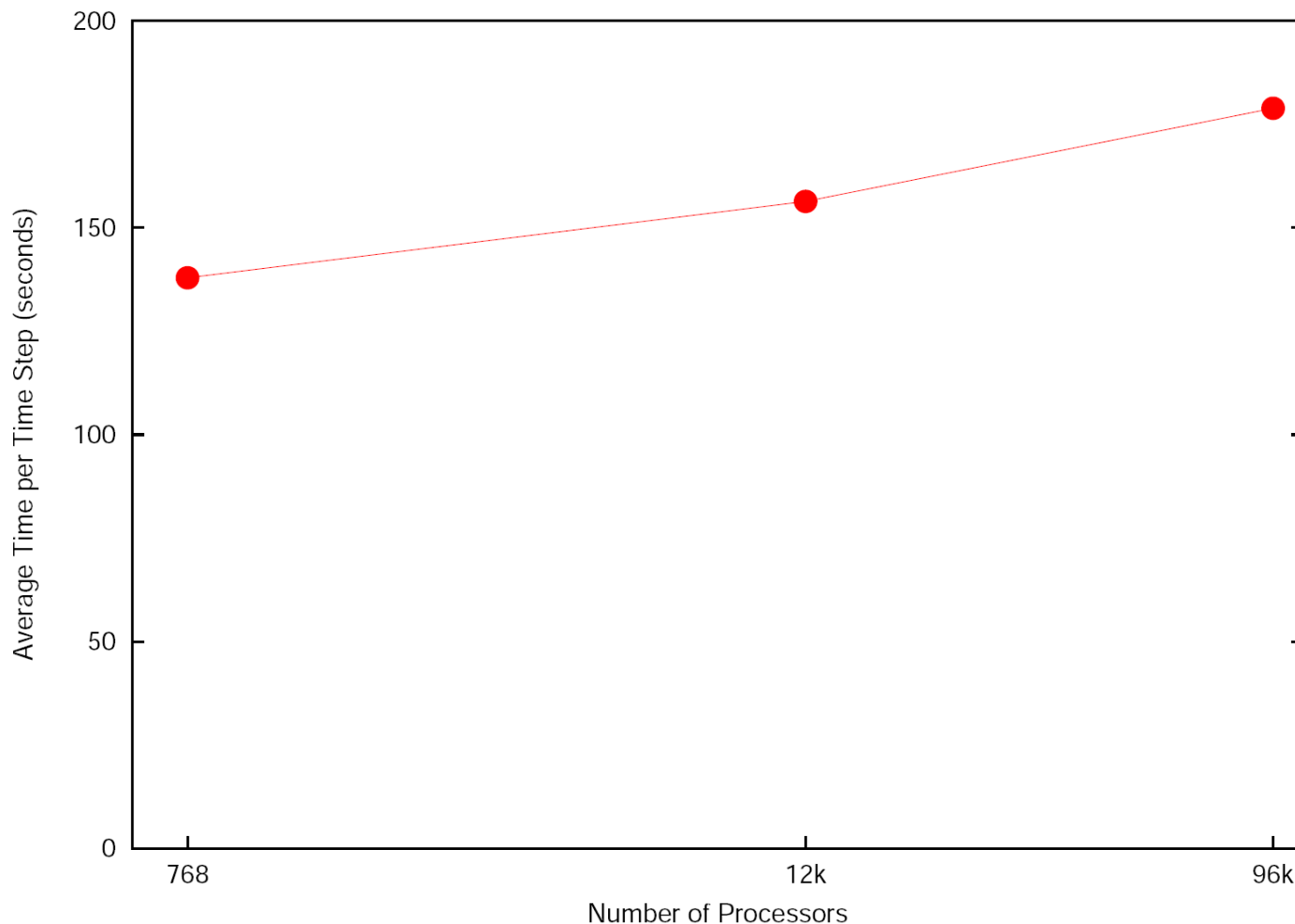
# Parallelization Strategy

- Hybrid MPI/OpenMP approach to parallelization.
  - Nodes assigned to grids, threads spawned on cores to work on grids



- Allows scaling to a factor of *nthreads* greater cores than pure MPI due to reduced communication time

- Weak scaling results for a 2-level Type Ia supernova simulation
  - Performed on jaguar at OLCF; 12 threads per MPI process
  - Each MPI process was assigned to a single  $128^3$  grid at each level
    - 768 processor simulation uses effective  $1024^3$  resolution
    - 96,000 processor simulation has effective  $5120^3$  resolution



Weak scaling  
results using a  
different number  
of threads per  
core would scale  
the numbers on  
the x-axis



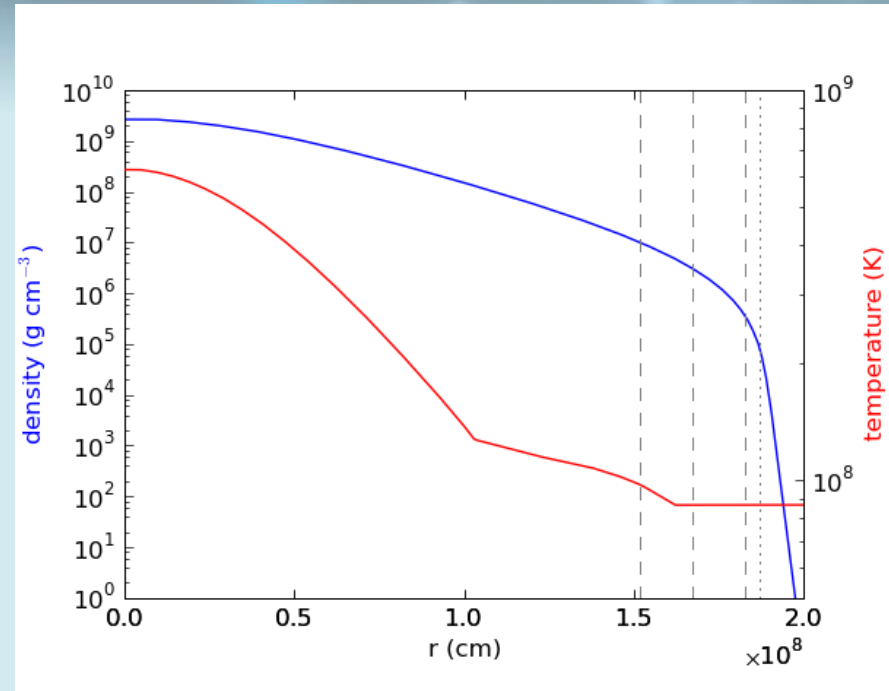
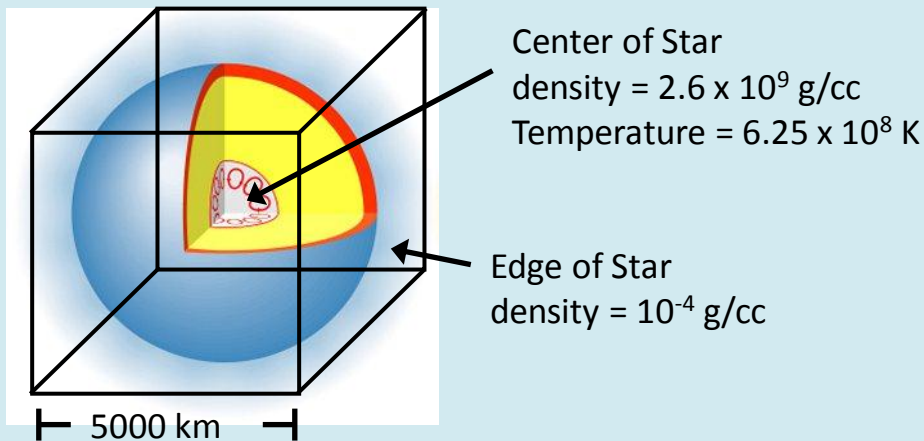


# **MAESTRO: Low Mach Number Astrophysics**

## **- Scientific Results**

# White Dwarf Convection: Initial Conditions

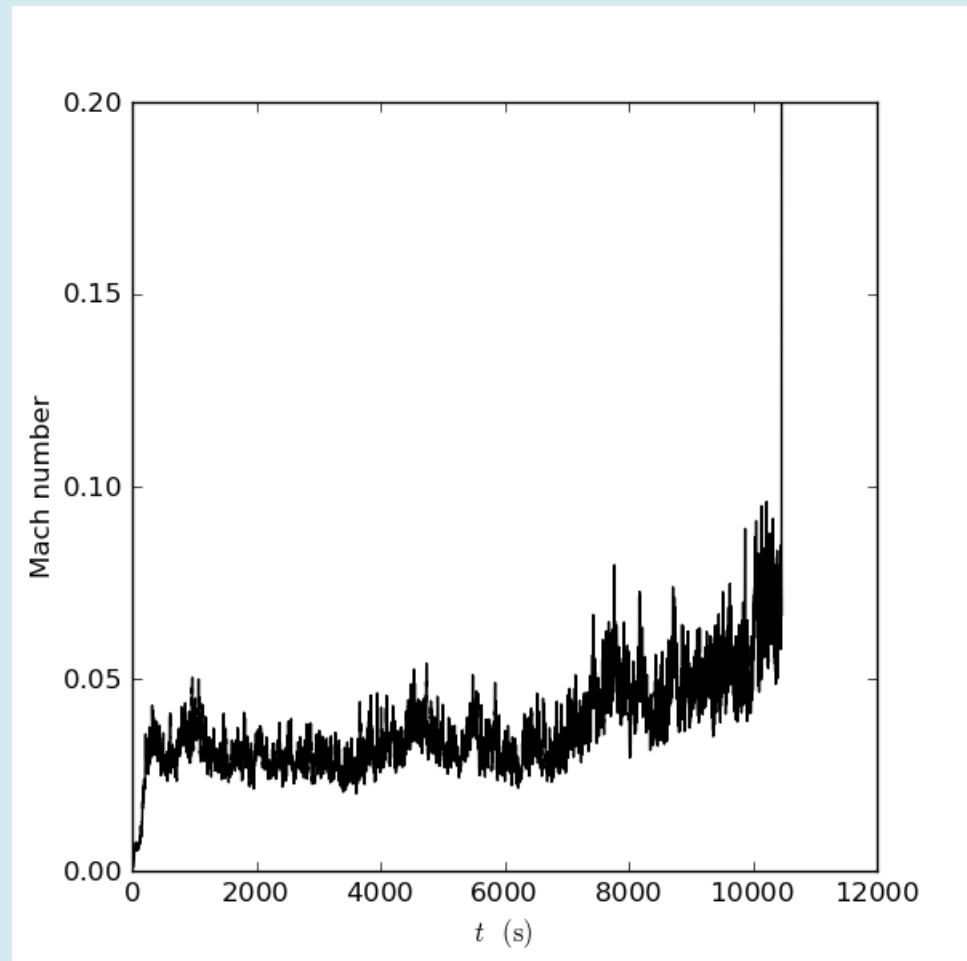
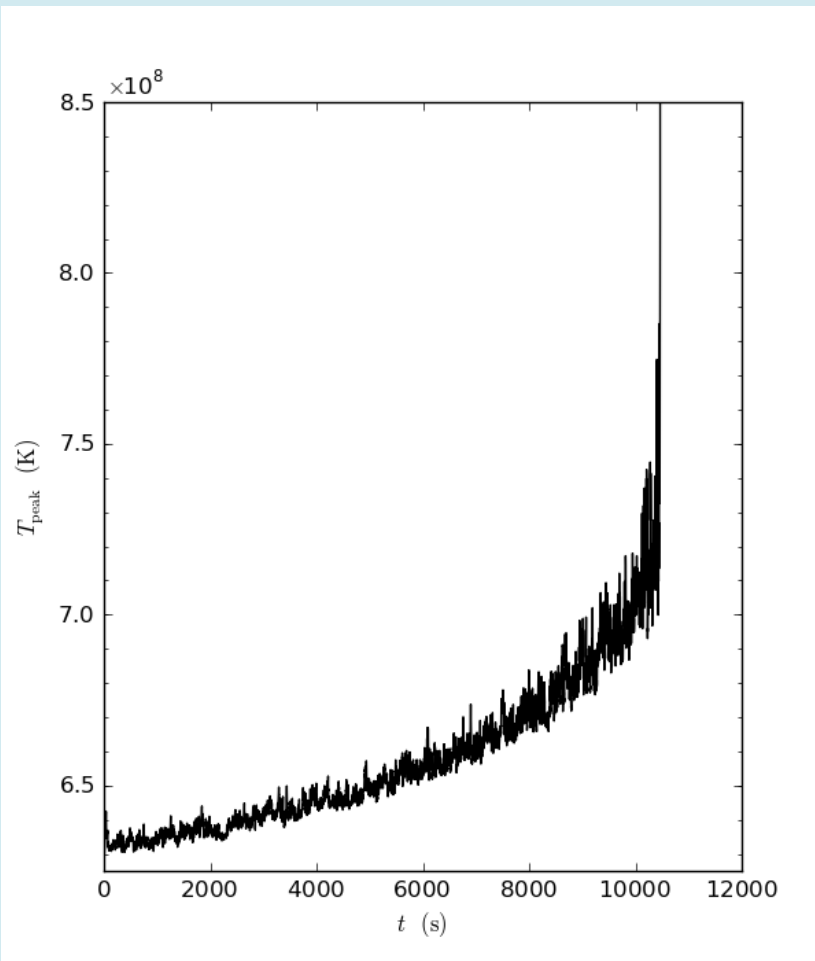
- Initial conditions
  - 1D KEPLER model mapped onto Cartesian grid
  - Random velocity perturbation added to prevent initial nuclear runaway



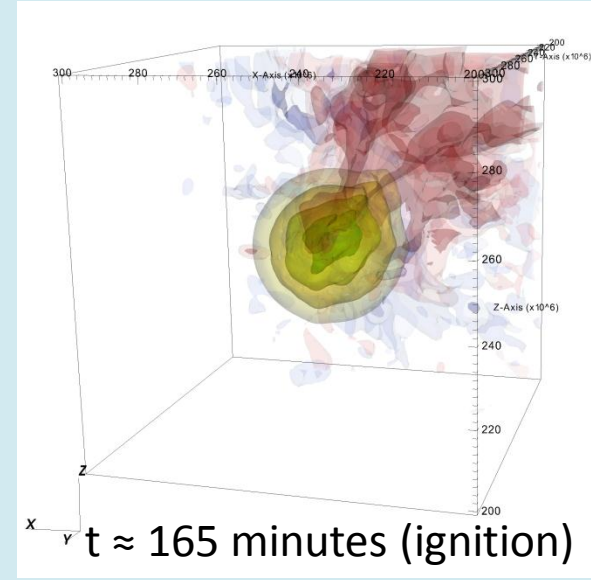
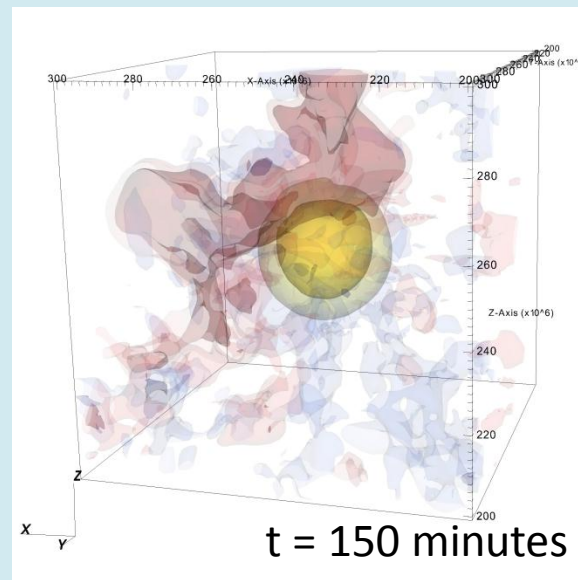
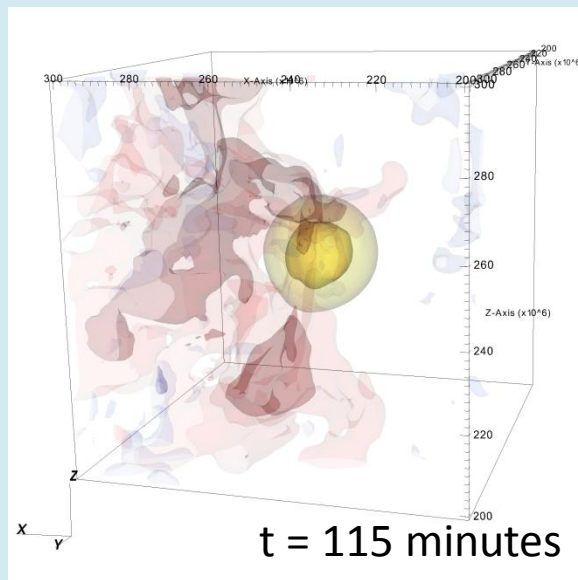
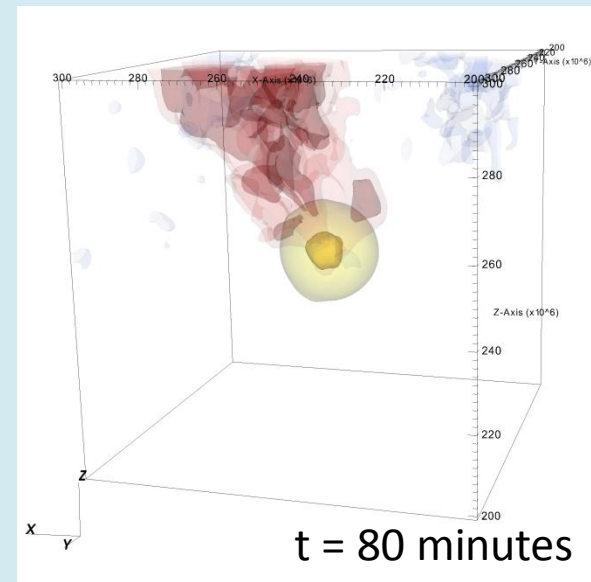
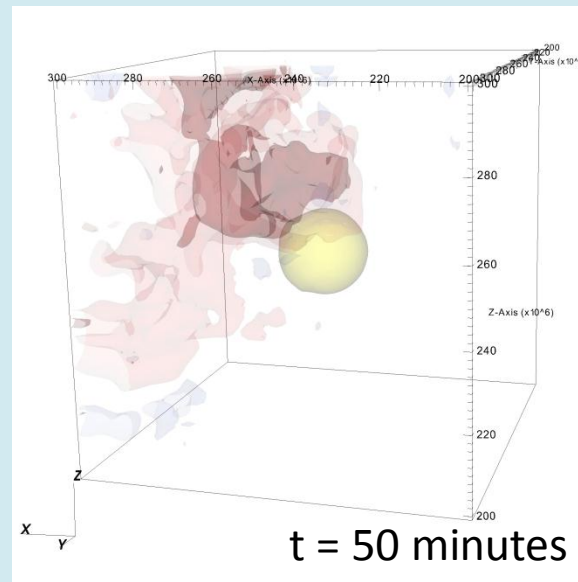
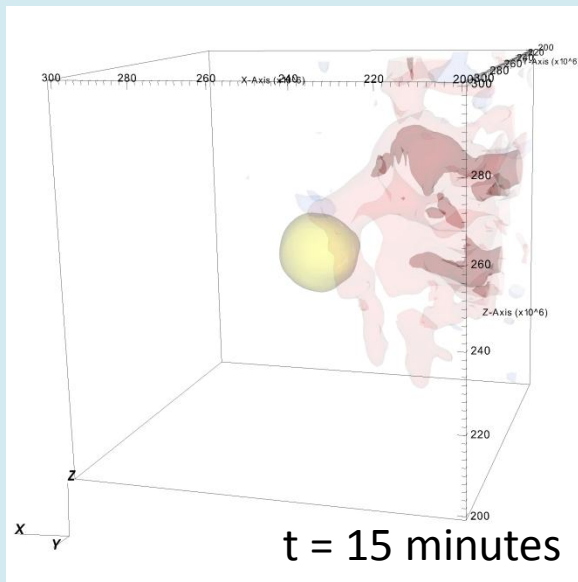
- Use 10K cores for 40 days (10 million CPU hours) to run effective  $1152^3$  resolution (4.3km zones) to ignition

# White Dwarf Convection: Long-Time Behavior

- Maximum temperature and Mach number vs. time

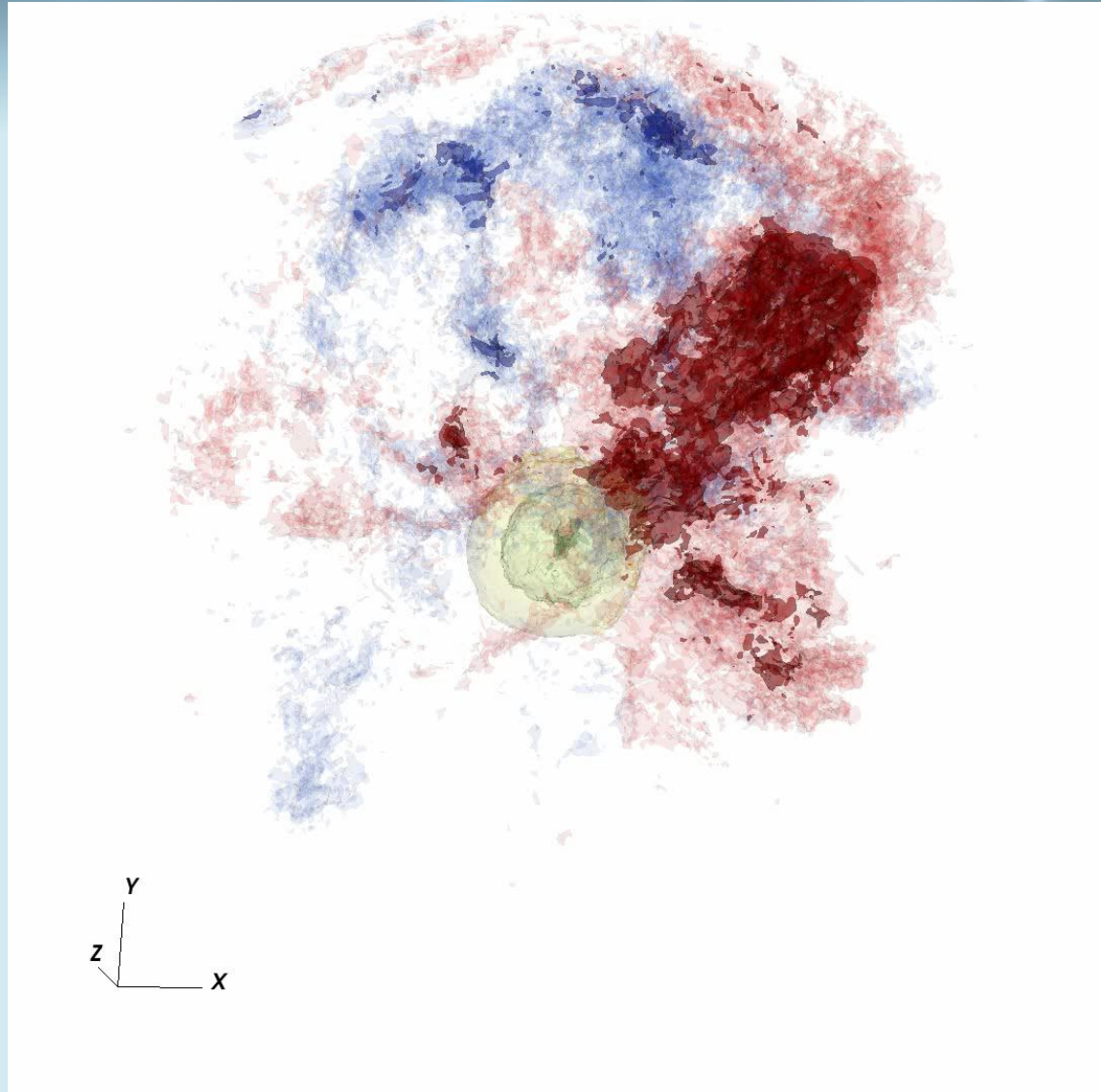


- Red / Blue = outward / inward radial velocity
- Yellow / Green = contours of increasing burning rate



# WD Convection: Ignition

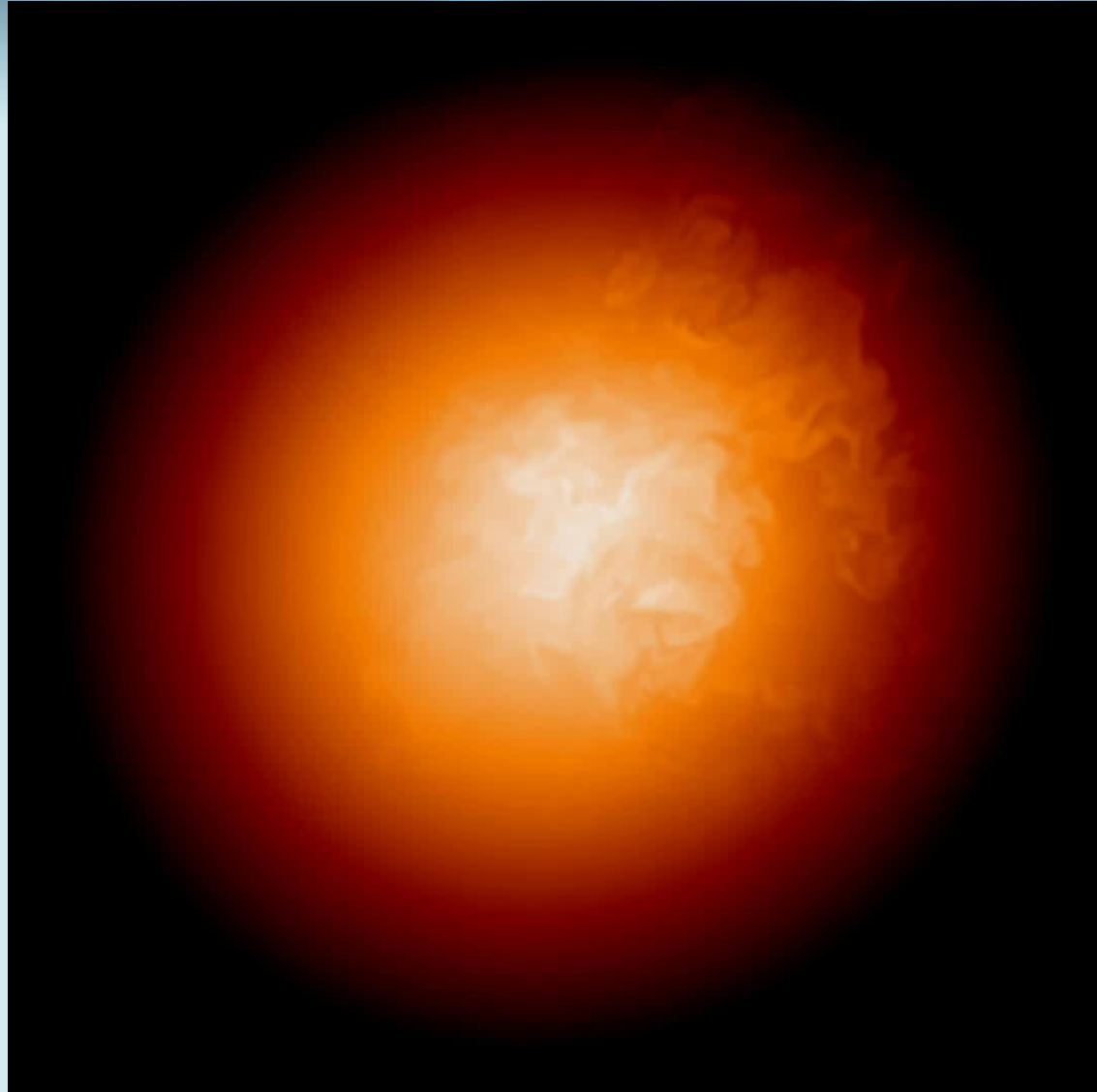
- Convective flow pattern a few minutes preceding ignition
  - Inner  $1000 \text{ km}^3$  of star
  - Effective  $2304^3$  resolution (2.2km) with 3 total levels of refinement
  - Red / Blue = outward / inward radial velocity
  - Yellow / Green = contours of increasing burning rate





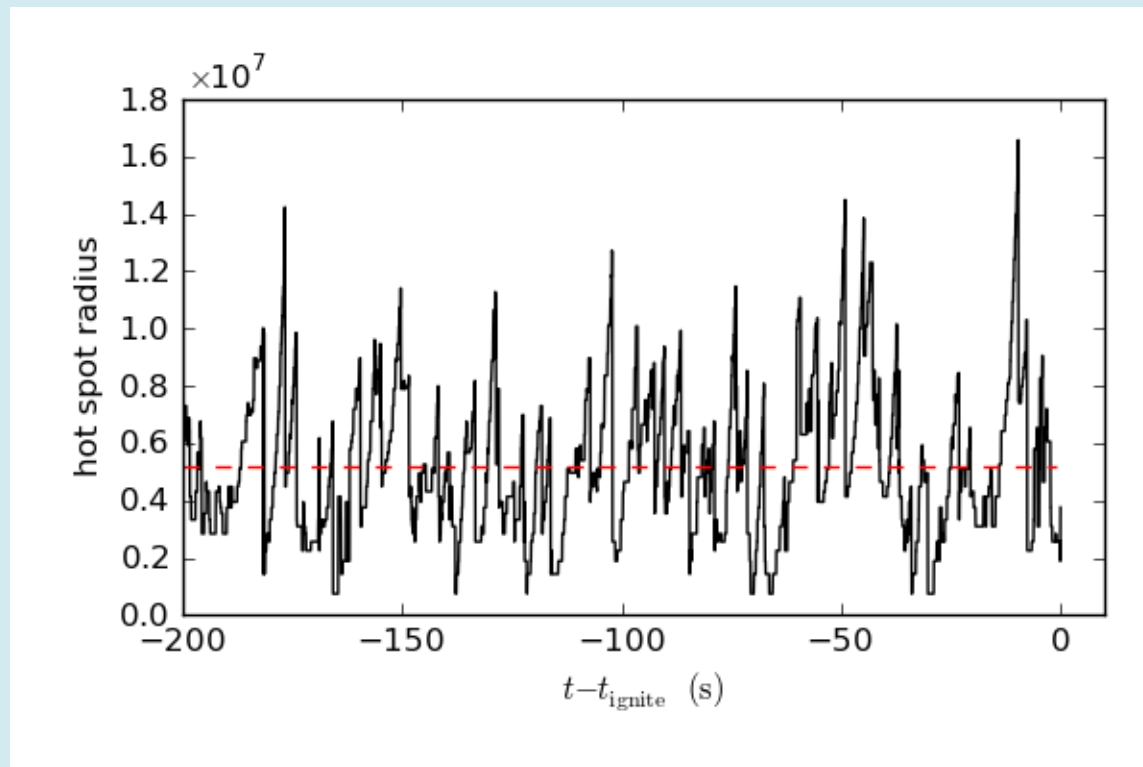
# WD Convection: Ignition

- Same data from the previous simulation
- 2D slice of temperature profile a few minutes preceding ignition



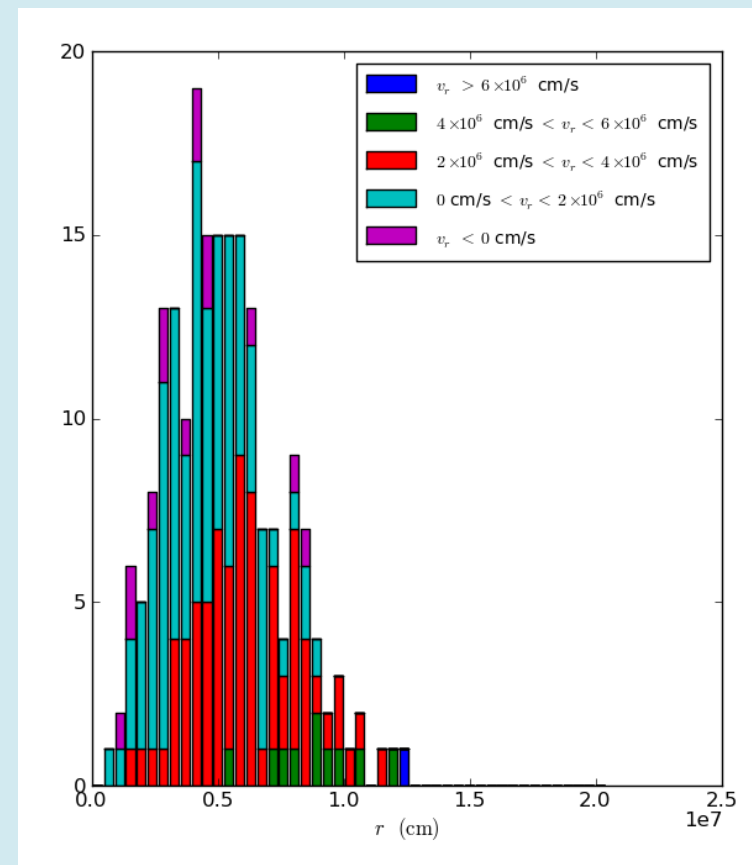
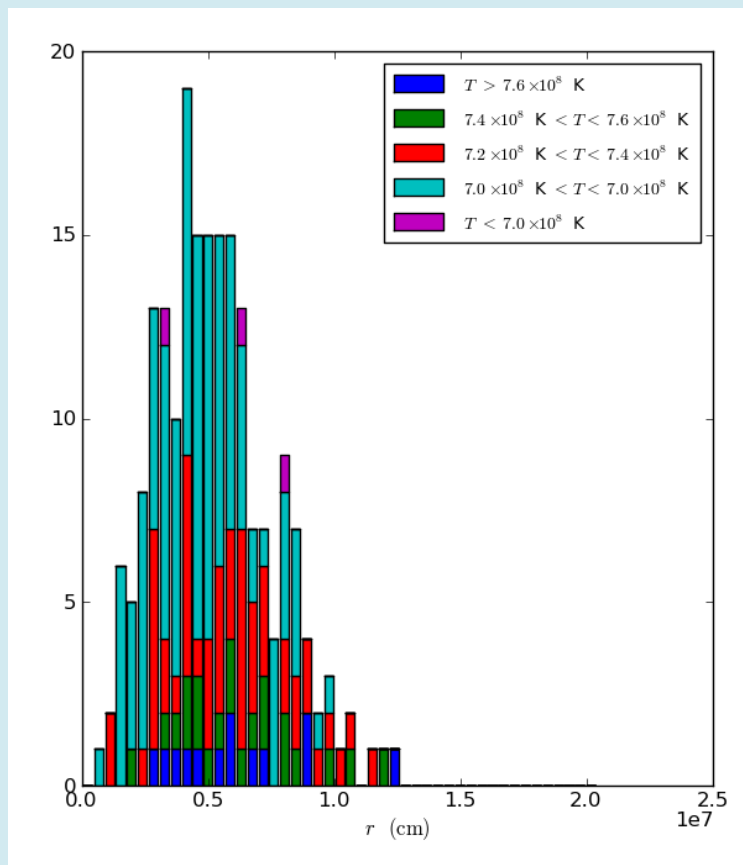
# WD Convection: Ignition

- Examining the radius of the hot spot over the last few minutes indicates ignition radius of 50-70 km off-center is favored.



# WD Convection: Ignition

- Histograms of ignition conditions over the final 200 seconds
  - (Left) Temperature and location of peak hot spot
  - (Right) Radial velocity and location of peak hot spot



# Summary / Future Work

- We have performed the first-ever full star simulations of convection preceding ignition in Type Ia supernovae
  - Low Mach number formulation
  - Adaptive mesh refinement
  - Performing science at 10K-20K cores, scaling to 100K cores
- What's next?
  - Examine the distribution of hot spots from our newest high-resolution studies.
  - Tracer particles to further understand development of hot spots
  - Examine role of turbulence and its effects on the first flames
  - Perform simulations in our compressible framework, CASTRO, using MAESTRO data as initial conditions