MAESTRO: An Adaptive Low Mach Number Hydrodynamics Algorithm for Stellar Flows

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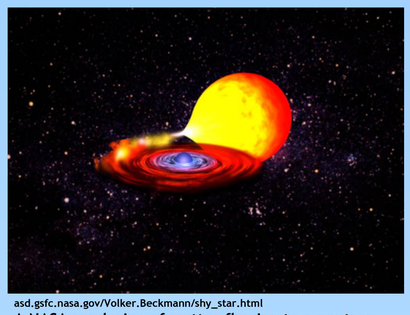
Many astrophysical phenomena are highly subsonic, requiring specialized numerical methods suitable for long-time integration. We present MAESTRO, a low Mach number stellar hydrodynamics code that can be used to simulate long-time, low-speed flows that would be prohibitively expensive to model using traditional compressible codes. MAESTRO is based on an equation set that we have derived using low Mach number asymptotics; this equation set does not explicitly track acoustic waves and thus allows a significant increase in the time step. MAESTRO is suitable for two- and three-dimensional local atmospheric flows as well as three-dimensional full-star flows, and uses adaptive mesh refinement (AMR) to locally refine grids in regions of interest. Our initial scientific applications include the convective phase of Type Ia supernovae and Type I X-ray Bursts on neutron stars.

What is MAESTRO?

MAESTRO is an adaptive, multidimensional hydrodynamics algorithm for low Mach number stellar flows. The Mach number is defined as the ratio of the fluid velocity to the speed of sound, $M = U / c_s$. Examples of such phenomena are the convection phases of Type Ia supernovae and Type I X-ray Bursts on neutron stars. The Mach number of such flows is small (~0.1), and requires a method suitable for long-time integration.

www.spacetelescope.org/images/html/opo9919i.html
SN1994D observed with the Hubble Space Telescope



asd.gsfc.nasa.gov/Volker.Beckmann/shy_star.html
A NASA rendering of matter flowing to a neutron star from a companion star, which can cause flare-ups known as x-ray bursts

Key features of the algorithm include:

- A pressure decomposition that filters sound waves, thereby allowing for long time integration
- A publicly available general stellar equation of state (Timmes)
- A hydrostatic, time-varying background / base state that captures large scale expansion and convection
- Adaptive mesh refinement (AMR) used to locally refine grids in regions of interest
- Massively parallel: scales well up to ~4,000 processors
- MAESTRO is suitable for local atmospheric flows as well as full-star flows

Low Mach Number Hydrodynamics

The fully compressible equations for stellar flows are not suitable for long-time integration due to the presence of acoustic sound waves. A low Mach number asymptotic expansion allows us decompose the pressure into a thermodynamic and dynamic component:

$$p(\mathbf{x}, r, t) = p_0(r, t) + \pi(\mathbf{x}, r, t); \quad \pi / p_0 = O(M^2).$$

affects only the thermodynamics — affects only the local dynamics

The thermodynamic pressure is in hydrostatic equilibrium with a background density:

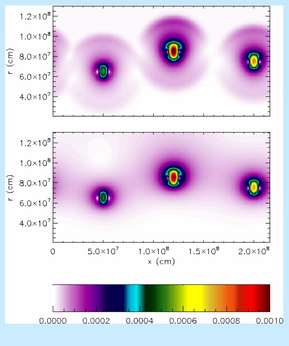
$$\nabla p_0(r, t) = -\rho_0(r, t) g \mathbf{e}_r$$

The low Mach number equation set uses this pressure decomposition:

Substitute $p = p_0 + \pi$

Conservation of momentum	$\frac{\partial \mathbf{U}}{\partial t}$	$+ \quad \mathbf{U} \cdot \nabla \mathbf{U}$	$=$	$-\frac{(\rho - \rho_0)}{\rho} g \mathbf{e}_r - \frac{1}{\rho} \nabla \pi$	
Conservation of mass	$\frac{\partial (\rho X_k)}{\partial t}$	$+ \quad \nabla \cdot (\rho X_k \mathbf{U})$	$=$	$\rho \dot{\omega}_k$	
Conservation of energy	$\frac{\partial (\rho h)}{\partial t}$	$+ \quad \nabla \cdot (\rho h \mathbf{U})$	$=$	$\rho H_{\text{nuc}} + \frac{Dp_0}{Dt} + \rho H_{\text{ext}} + \nabla \cdot \kappa \nabla T$	

p_0 replaces p



- We have "filtered out" the sound waves.
- The time step is based on the fluid velocity, not the sound speed, and increases by a factor of ~1/*M*.
- Mathematically, we now require an elliptic constraint on the velocity:

$$\nabla \cdot (\beta_0 \mathbf{U}) = \beta_0 \left(S - \frac{1}{\bar{\Gamma}_1 p_0} \frac{\partial p_0}{\partial t} \right);$$

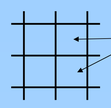
$$S = \sigma \sum_k \xi_k \dot{\omega}_k + \frac{1}{\rho p_\rho} \sum_k p_{X_k} \dot{\omega}_k + \frac{\sigma}{\rho} \nabla \cdot \kappa \nabla T + \sigma H_{\text{nuc}} + \sigma H_{\text{ext}}$$

Above is a plot of the Mach number for a set of rising hot bubbles in a white dwarf environment using a compressible code (above) and MAESTRO (below). In the MAESTRO simulation, acoustic equilibration happens infinitely fast.

The one-dimensional background state (ρ0 and p0) represents the "average" state of the star, and evolves in response to heating and convective overturning. We use advanced geometric mapping techniques to couple the background state with the full state.

Numerical Procedure

We discretize our domain into regular Cartesian cells and represent our state variables as averages over each cell.



$\mathbf{U}, \rho, X_k, h, T, \pi, \rho_0, p_0$

$$\begin{aligned}
\frac{\partial \mathbf{U}}{\partial t} &+ \mathbf{U} \cdot \nabla \mathbf{U} &&= -\frac{(\rho - \rho_0)}{\rho} g \mathbf{e}_r - \frac{1}{\rho} \nabla \pi \\
\frac{\partial (\rho X_k)}{\partial t} &+ \nabla \cdot (\rho X_k \mathbf{U}) &&= \rho \dot{\omega}_k \\
\frac{\partial (\rho h)}{\partial t} &+ \nabla \cdot (\rho h \mathbf{U}) &&= \rho H_{\text{nuc}} + \frac{Dp_0}{Dt} + \rho H_{\text{ext}} + \nabla \cdot \kappa \nabla T \\
\nabla \cdot (\beta_0 \mathbf{U}) &= \beta_0 \left(S - \frac{1}{\bar{\Gamma}_1 p_0} \right)
\end{aligned}$$

We use operator splitting to advance the solution in time. We advance the reaction terms by Δt/2, the advection/diffusion terms by Δt, and the reaction terms by Δt/2.

We use a stiff ODE solver (VODE) to compute the reaction terms.

We solve the hyperbolic part of the equations with a second-order Godunov method, in which we compute time-centered interface states used to discretize the advection terms. We use the unsplit piecewise-parabolic method (Colella) to construct edge states.

We solve a semi-implicit discretization for the thermal diffusion using geometric multigrid with Gauss-Seidel relaxation.

We use a fractional step approach for velocity, in which we impose the elliptic constraint while simultaneously solving for the dynamic pressure update. This is based on the projection method of Bell, et al.

The background state also evolves in time in response to heating and large scale convection:

$$\frac{\partial p_0}{\partial t} = -w_0 \frac{\partial p_0}{\partial r} + \eta_\rho g; \qquad \frac{\partial \rho_0}{\partial t} = -\frac{\partial (\rho_0 w_0)}{\partial r} - \frac{\partial \eta_\rho}{\partial r},$$

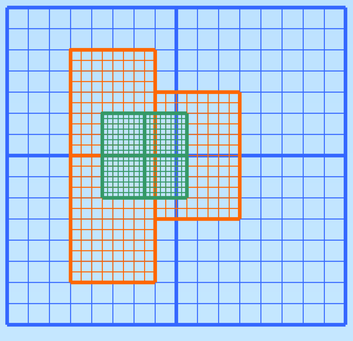
Expansion velocity computed by averaging the heating in the star — Mixing term captures large scale convection and ensures the background state represents the "average" state of the star

AMR

Our AMR approach uses a nested hierarchy of logically rectangular grids with successively finer grids at each level.

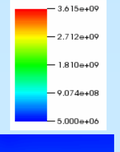
A user-specified tagging routine indicates where higher resolution is desired.

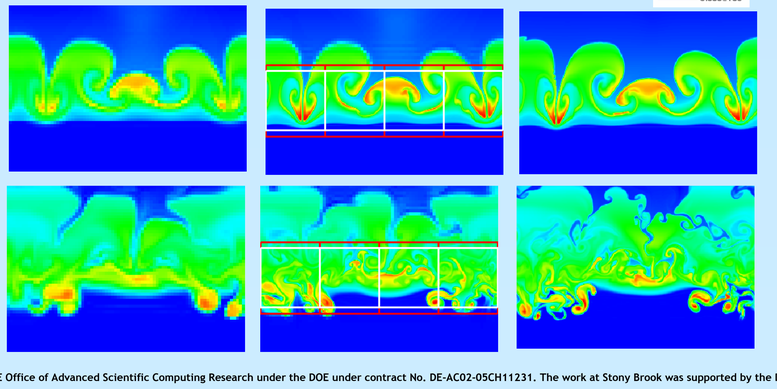
We parallelize our code distributing the grids on each level across processors.



Example: Forced Convection

We examine the convection profiles of a white-dwarf environment subject to an externally prescribed heating layer. Below are plots of the temperature profiles after 3 seconds (above) and 4 seconds (below) of external heating. The left column is a single-level coarse simulation. The center column is a multi-level solution with 2 levels of refinement. The right column is a single-level fine simulation. The multi-level solution is able to capture the finer-scale structure not visible in the coarse solution.



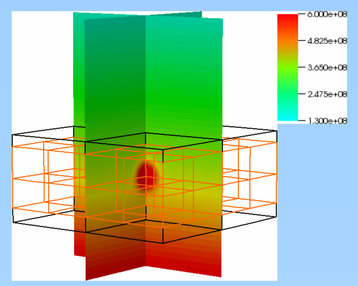


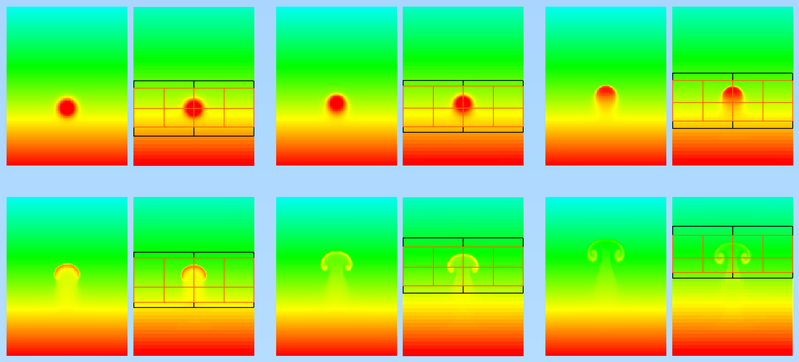
Example: Bubble Rise

We examine the dynamics of a hot bubble rising in a white dwarf environment using a single-level simulation, and an adaptive simulation with two levels of refinement.

On the right, we show a 3D view of the temperature and the initial adaptive grid structure. The colored boxes indicate grids are varying levels of refinement.

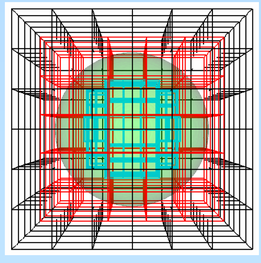
Below, we show a time-lapse cross section at 0.5 second time intervals, showing that the adaptive solution captures the same dynamics as the single-level solution.



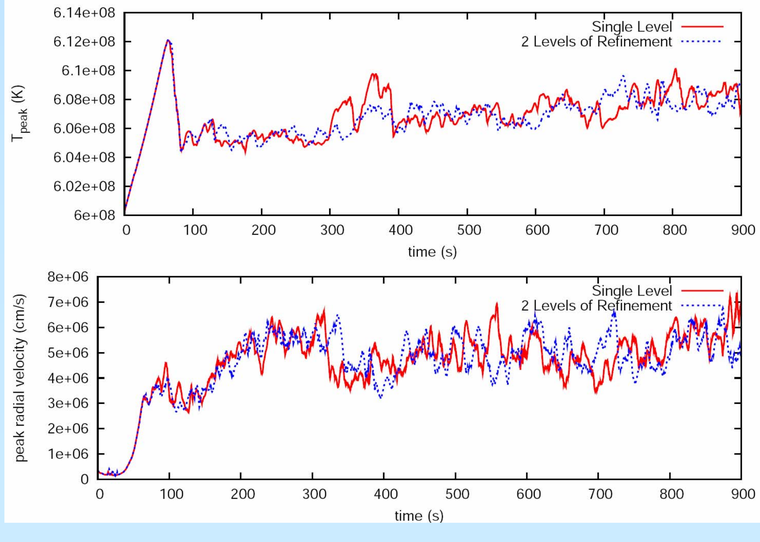


Example: Full-Star Convection

We perform full star calculations simulating the convective phase of a Type Ia supernova with and without AMR. An adaptive case with two levels of refinement requires 18 seconds per time step, and the single level case requires 36 seconds per time step using 216 processors on Franklin at NERSC. The initial adaptive grid structure is shown on the right. The colored boxes indicate grids at different levels of refinement.



Below is a plot of maximum temperature and peak radial velocity as a function of time, comparing the single level calculation from our previous publication to an adaptive grid calculation with 2 levels of refinement.



Scientific Applications

For Type Ia supernovae convection results, visit Mike Zingale's poster (Zingale et al., ApJ 704, 2009)

For Type I X-ray Burst results, visit Chris Malone's poster.

For further algorithmic details, see Nonaka et al., submitted to ApJS.

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