Zero Mach # single component flow

(1) $$\rho_t + \nabla\cdot\rho u = 0$$

(2) $$(\rho U)_t + \nabla\cdot\rho u u + \nabla\pi = \nabla\cdot\tau \qquad \tau = \mu\left(\nabla U + \nabla U^T\right) - \frac{2}{3}\mu(\nabla\cdot U)I$$

(3) $$(\rho h)_t + \nabla\cdot\rho u h = \nabla\cdot\lambda\nabla T + \frac{Dp}{Dt}$$

(4) $$p = \rho R T$$

$$h = h(T) = \int^T c_p(\tau)\,d\tau$$

$$\frac{\partial h}{\partial T} = c_p(T) \;..$$

For an open container $p = p_0$ fixed

more generally in zero Mach # limit $p = p(t)$

From 3.

$$\rho\frac{Dh}{Dt} = \cancel{\rho c_p}\;\rho c_p\frac{DT}{Dt} = \nabla\cdot\lambda\nabla T + \frac{Dp}{Dt}.$$

From 4 $$\frac{Dp}{Dt} = RT\frac{D\rho}{DT} + R\rho\frac{DT}{Dt}$$

$$= RT(-\rho\nabla\cdot U) + \frac{R\rho}{\rho c_p}\left(\nabla\cdot\lambda\nabla T + \frac{Dp}{Dt}\right)$$

$$p\,\nabla\cdot U = \frac{R\rho}{\rho c_p}(\nabla\cdot\lambda\nabla T) + \left(\frac{R\rho}{\rho c_p} - 1\right)\frac{Dp}{Dt}$$

For constant γ, γ-law gas this becomes.

$$(5) \qquad \nabla\cdot U = \frac{\gamma-1}{\gamma p}\nabla\cdot\lambda\nabla T - \frac{1}{\gamma p}\frac{Dp}{Dt} \equiv S$$

In a closed chamber $p = p(t)$

$$0 = \int_\partial u\cdot n = \int \nabla\cdot U = \frac{\gamma-1}{\gamma p}\int_\partial \lambda\frac{\partial T}{\partial n} - \frac{1}{\gamma p}\int\frac{\partial p}{\partial t}\,d\sigma$$

$$= \frac{\gamma-1}{\gamma p}\int_\partial \lambda\frac{\partial T}{\partial n} - \frac{1}{\gamma p}p'(t)\,V$$

volume of chamber.

$$\frac{1}{\gamma p}\frac{\partial p}{\partial t} = \frac{\gamma-1}{\gamma p}\frac{1}{V}\int_\partial \lambda\frac{\partial T}{\partial n}$$ defines ~~p~~ thermodynamic pressure.

(5) then satisfies ~~the~~ integrability condition for a projection; e.g., $\int S = 0$.

As noted above, in an open chamber $p = p_0$.

so $\nabla\cdot U = \frac{\gamma-1}{\gamma p}\nabla\cdot\lambda\nabla T.$

Aside: Rupert has a way to impose constraint that exactly keeps you on the constraint but it is not ~~gen~~ general so we will try to avoid that even though it works here.

Note also he views energy equation as defining the constraint.

For the open case

Momen:

$$U_t + U\cdot\nabla U + \frac{1}{\rho}\nabla\pi = \ldots$$

$$\nabla\cdot U = \frac{\gamma-1}{\gamma p}\nabla\cdot\lambda\nabla T \equiv S.$$

Want decomposition

$$V = U_d + \frac{1}{\rho}\nabla\phi + \nabla\xi \qquad \text{where you want } U = U_d + \nabla\xi.$$

where $\nabla\cdot U_d = 0$

$\Delta\xi = S$

$$\int\left(U_d\cdot\frac{1}{\rho}\nabla\phi\right)\rho\, dm = 0.$$

given ρ define $\mathbb{P}_\rho$ as ρ-weighted projection.

$$U_d = \mathbb{P}_\rho(V - \nabla\xi)$$

$$U = \mathbb{P}_\rho(V - \nabla\xi) + \nabla\xi.$$

Can be computed directly from

$$\nabla\cdot V = \nabla\cdot\frac{1}{\rho}\nabla\phi + \Delta\xi$$

or $$\nabla\cdot\frac{1}{\rho}\nabla\phi = \nabla\cdot V - S.$$

Rmks 1: We ~~play~~ add a wrinkle to reduce drift of EOS. (avoided in Rupert form)

2: Closed container is similar but with added ODE for pressure

Quest: How can we fit this into SDC framework.

Now, for completeness what does Rupert do.

for a constant γ-law gas

$$p = (\gamma-1)\rho e$$

~~[illegible]~~

$$h = e + p/\rho$$

$$\rho h = \rho e + p$$

$$= p\left(\frac{1}{\gamma-1} + 1\right)$$

$$= \left(\frac{\gamma}{\gamma-1}\right) p$$

So 3 becomes

$$\frac{\gamma}{\gamma-1}\left(p_t + \nabla\cdot u p\right) = \nabla\cdot\lambda\nabla T + \frac{Dp}{Dt}$$

if p is constant in space then

$$-\left(\frac{\gamma-1}{\gamma} - 1\right)\frac{\partial p}{\partial t} + p\nabla\cdot u = \frac{\gamma-1}{\gamma}\nabla\cdot\lambda\nabla T$$

$$+\frac{1}{\gamma}\frac{\partial p}{\partial t} + p\nabla\cdot u = \frac{\gamma-1}{\gamma}\nabla\cdot\lambda\nabla T$$

This says ~~[illegible]~~ constraint from diff EOS is equivalent to what you get from Energy eqtn in this case.

Rupert then has a way to stay on constraint discretely.